accuracy for the considered complex simulated examples, including rotating and vibrating target parts.

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Robust Adaptive PID Tracking Control Design for Uncertain Spacecraft Systems: A Fuzzy Approach

In this paper, a robust adaptive PID-type controller incorporating a fuzzy logic system and a sliding-mode control action for robust H_∞ tracking performance is developed in spacecraft attitude maneuvers under parameter uncertainties and external disturbances. The fuzzy logic system is used to compensate the parameter uncertainties. The robust adaptive PID-type tracking control problems are characterized in terms of eigenvalue problem (EVP). The EVP can be efficiently solved by the LMI toolbox in Matlab. The proposed methods are simple and the PID parameters can be obtained systematically. Simulation results indicate that the desired performance for the tracking control schemes of the uncertain spacecraft systems can be achieved using the proposed methods.

NOMENCLATURE

J Inertia matrix

h Total spacecraft angular momentum in body axes, $h = [h_1\ h_2\ h_3]^T = J\omega$

θ Attitude Euler angle, $\theta = [\theta_1\ \theta_2\ \theta_3]^T$

$[\nu \times]$ Cross product matrix associated with the vector $\nu = [\nu_1 \ \nu_2 \ \nu_3]^T$ where

$$[\nu \times] = \begin{bmatrix} 0 & -\nu_3 & \nu_2 \\ \nu_3 & 0 & -\nu_1 \\ -\nu_2 & \nu_1 & 0 \end{bmatrix}$$

τ Torque vector due to actuator,
$$\tau = [\tau_{a_1} \ \tau_{a_2} \ \tau_{a_3}]^T$$

τ'_d External disturbance torque vector

ω General angular velocity vector in body axes, $\omega = [\omega_1 \ \omega_2 \ \omega_3]^T$.

I. INTRODUCTION

The attitude tracking control of spacecraft system has received extensive attention in recent decades, and several methods of spacecraft attitude tracking control have been developed to treat this problem. The tracking control design of nonlinear spacecraft attitude is a difficult process, and in practical control systems the plants are always nonlinear and uncertain. Thus many nonlinear control methods have been developed for spacecraft attitude tracking to overcome the difficulty in controller design for a real system. Based on linearization using coordinate transformation and nonlinear feedback, a controller for attitude control has been derived [1]. The feasibility of applying the feedback linearization technique to spacecraft attitude control and momentum management problem has also been discussed [2]. A sliding-mode [3] approach has also been used for spacecraft attitude control. The robust backstepping control for spacecraft attitude maneuvers has been discussed [4]. More relevant to this study, optimal control theory has been used for attitude control of spacecraft system [5]. The approach of H_∞ control has been applied to the Space Station attitude and momentum control problem while taking into consideration the linearized equations of motion [6]. Recent, a mixed H_2/H_∞ control design has been developed to treat the spacecraft attitude control problem under parameter perturbation and external noise [7, 8].

In this paper, a robust adaptive PID-type tracking controller incorporating a fuzzy logic system and a sliding-mode control action for robust H_∞ tracking performance is developed in spacecraft attitude control under parameter uncertainties and external disturbances. In this study, the PID parameters can be obtained systematically according to desired performances. In practical spacecraft systems, uncertainties which may effect the tracking performance are inevitable. A fuzzy logic system is introduced here to learn the uncertainties by an adaptive algorithm, that is, the fuzzy logic system is used to compensate for the parameter uncertainties. Actually, the fuzzy logic system

can only approximate the parameter uncertainties. Approximation errors between fuzzy logic system and parameter uncertainties do exist. Therefore, a sliding-mode control action is included to eliminate the effect of approximation error. The aim of this study is to find a robust H_∞ adaptive PID-type controller incorporating a fuzzy logic system such that the closed-loop spacecraft systems are stable.

The paper is organized as follows. The model description and problem formulation is presented in Section II. In Section III, PID control with robust H_∞ tracking performance is presented. In Section IV, a simulation example is provided to demonstrate the design procedures. Concluding remarks are made in Section V.

II. MODEL DESCRIPTION AND PROBLEM FORMULATION

The mathematical model of a spacecraft, treated as a rigid body, is commonly composed of two sets of equations called the attitude kinematic equation and the spacecraft dynamic equation which can be described, respectively, as follows [9]:

Attitude kinematics:
$$\omega = R(\theta)\dot{\theta} - \omega_c(\theta) \tag{1}$$

Spacecraft dynamics:
$$J\dot{\omega} = [h\times]\omega + 3\omega_0^2[c\times]Jc + \tau_a + \tau'_d \tag{2}$$

where

$$R(\theta) = \begin{bmatrix} 1 & 0 & -\sin\theta_2 \\ 0 & \cos\theta_1 & \sin\theta_1\cos\theta_2 \\ 0 & -\sin\theta_1 & \cos\theta_1\cos\theta_2 \end{bmatrix}$$

$$\omega_c(\theta) = \omega_0 \begin{bmatrix} \cos\theta_2\sin\theta_3 \\ \cos\theta_1\cos\theta_3 + \sin\theta_1\sin\theta_2\sin\theta_3 \\ \cos\theta_1\sin\theta_2\sin\theta_3 - \sin\theta_1\cos\theta_3 \end{bmatrix}$$

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$$

$$h = J\omega$$

$$c = \begin{bmatrix} -\sin\theta_2 \\ \sin\theta_1\cos\theta_2 \\ \cos\theta_1\cos\theta_2 \end{bmatrix}.$$

Differentiating (1), we have

$$\dot{\omega} = R(\theta)\ddot{\theta} + \left(\frac{dR(\theta)}{dt}\right)\dot{\theta} - \left(\frac{d\omega_c(\theta)}{dt}\right). \tag{3}$$

Substituting ω and $\dot{\omega}$ from (1) and (3) into (2), we obtain

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta,\dot{\theta}) = \tau + \tau'_d \tag{4}$$

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where

$$M(\theta) = JR(\theta)$$

$$C(\theta,\dot{\theta}) = J\frac{dR(\theta)}{dt} - [h\times]R(\theta)$$

$$G(\theta,\dot{\theta}) = -J\frac{d\omega_c(\theta)}{dt} + [h\times]\omega_c(\theta) - 3\omega_0^2[c\times]Jc$$

$$\tau = [\tau_{a_1}\ \ \tau_{a_2}\ \ \tau_{a_3}]^T$$

$$\tau_d' = [\tau_{d_1}\ \ \tau_{d_2}\ \ \tau_{d_3}]^T.$$

REMARK 1   This description of (1) is defined for all $(\theta_1,\theta_2,\theta_3)$ expect $\theta_2 = \pm(\pi/2)(2n+1)$ for any integer n. This singularity (i.e., the determinant of the matrix $R(\theta)$) becomes zero at $\theta_2 = \pm(\pi/2)(2n+1)$ arises owing to the choice of the set of rotations which define the orientation corresponding to the singularity $\theta_2 = \pm\pi/2$ lies in the control region of attitude angles, another set of rotations can be defined to eliminate this singularity. In this paper, we are interested in the trajectories in the bounded region Ω.

In practical spacecraft systems, however, perturbations in system parameters that are due to the flexible structure, plant uncertainties, or parameter variations, and the change of the orientation of solar arrays on the spacecraft are inevitable. Thus, the inertia matrix can be rewritten as

$$J = J_0 + \Delta J. \tag{5}$$

Therefore the system parameter matrices $M(\theta)$, $C(\theta,\dot{\theta})$ and $G(\theta,\dot{\theta})$ in spacecraft model in (4) can be divided into nominal parts and perturbed parts as follows:

$$M(\theta) = M_0(\theta) + \Delta M(\theta)$$

$$C(\theta,\dot{\theta}) = C_0(\theta,\dot{\theta}) + \Delta C(\theta,\dot{\theta})$$

$$G(\theta,\dot{\theta}) = G_0(\theta,\dot{\theta}) + \Delta G(\theta,\dot{\theta}) \tag{6}$$

$$[h\times] = [h_0\times] + [\Delta h\times]$$

where

$$M_0(\theta) = J_0R(\theta)$$

$$C_0(\theta,\dot{\theta}) = J_0\frac{dR(\theta)}{dt} - [h_0\times]R(\theta)$$

$$G_0(\theta,\dot{\theta}) = -J_0\frac{d\omega_c(\theta)}{dt} + [h_0\times]\omega_c(\theta) - 3\omega_0^2[c\times]J_0c$$

$$h_0 = J_0\omega$$

and

$$\Delta M(\theta) = \Delta JR(\theta)$$

$$\Delta C(\theta,\dot{\theta}) = \Delta J\frac{dR(\theta)}{dt} - [\Delta h\times]R(\theta)$$

$$\Delta G(\theta,\dot{\theta}) = -\Delta J\frac{d\omega_c(\theta)}{dt} + [\Delta h\times]\omega_c(\theta) - 3\omega_0^2[c\times]\Delta Jc$$

$$\Delta h = \Delta J\omega.$$

Therefore, the motion equations of the spacecraft manipulator in (4) can be expressed as follows:

$$(M_0(\theta) + \Delta M(\theta))\ddot{\theta} + (C_0(\theta,\dot{\theta}) + \Delta C(\theta,\dot{\theta}))\dot{\theta}$$
$$+ (G_0(\theta,\dot{\theta}) + \Delta G(\theta,\dot{\theta})) = \tau + \tau_d'. \tag{7}$$

Let us consider the following control law

$$\tau(t) = M_0(\theta)[\ddot{\theta}_d + \tau_{\text{pid}}(t) + \tau_s(t) - \tau_f(\eta,\Xi)]$$
$$+ C_0(\theta,\dot{\theta})\dot{\theta} + G_0(\theta,\dot{\theta}) \tag{8}$$

where $\tau_{\text{pid}}(t)$ is a PID-type control action to be designed; $\tau_s(t)$ is a sliding-mode control to be designed; $\tau_f(\eta,\Xi)$ is a fuzzy logic system to be specified; $\theta_d(t)$ denotes the desired trajectory vector which is assumed to be continuously differentiable (at least C^2).

By substituting (8) into (4), we obtain

$$\frac{d^2\tilde{\theta}(t)}{dt^2} = -M_0^{-1}(\theta)[\Delta M(\theta)\ddot{\theta} + \Delta C(\theta,\dot{\theta})\dot{\theta} + \Delta G(\theta,\dot{\theta})]$$
$$+ \tau_{\text{pid}}(t) + \tau_s(t) - \tau_f(\eta,\Xi) + M_0^{-1}(\theta)\tau_d' \tag{9}$$

where $\tilde{\theta}(t)$ is defined as tracking error, i.e.,

$$\tilde{\theta}(t) \overset{\Delta}{=} \theta(t) - \theta_d(t). \tag{10}$$

If we define

$$e(t) \overset{\Delta}{=} \begin{bmatrix} \tilde{\theta}_I(t) \\ \tilde{\theta}(t) \\ \dfrac{d\tilde{\theta}(t)}{dt} \end{bmatrix} = \begin{bmatrix} \int(\theta(t) - \theta_d(t))dt \\ \theta(t) - \theta_d(t) \\ \dot{\theta}(t) - \dot{\theta}_d(t) \end{bmatrix} \tag{11}$$

$$\Delta f(\eta) \overset{\Delta}{=} -M_0^{-1}(\theta)[\Delta M(\theta)\ddot{\theta} + \Delta C(\theta,\dot{\theta})\dot{\theta} + \Delta G(\theta,\dot{\theta})]$$

$$\tag{12}$$

and

$$\tau_d(t) \overset{\Delta}{=} M_0^{-1}(\theta)\tau_d'(t) \tag{13}$$

with $\eta = [\theta(t),\dot{\theta}(t),\ddot{\theta}(t)]^T$ and $\tilde{\theta}_I(t)$ is defined as integration of tracking error, i.e.,

$$\tilde{\theta}_I(t) \overset{\Delta}{=} \int \tilde{\theta}(t)dt = \int(\theta(t) - \theta_d(t))dt \tag{14}$$

then the tracking error dynamic equations can be expressed as follows:

$$\dot{e}(t) = Ae(t) + B\tau_{\text{pid}}(t) + B\tau_s(t) + B(\Delta f(\eta) - \tau_f(\eta,\Xi))$$
$$+ B\tau_d(t) \tag{15}$$

where

$$A = \begin{bmatrix} 0_{3\times 3} & I_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & I_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \end{bmatrix}$$

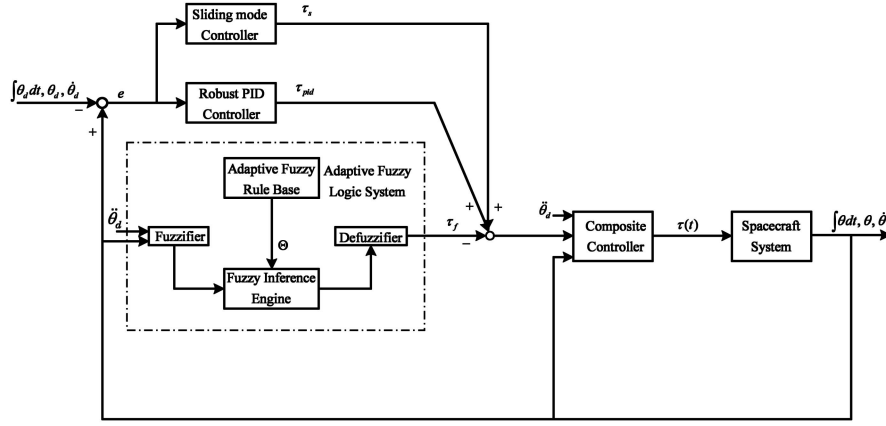$$B = \begin{bmatrix} 0_{3\times 3} \\ 0_{3\times 3} \\ I_{3\times 3} \end{bmatrix}.$$

Fig. 1.   Fuzzy logic system for spacecraft tracking control.

Let us specify the PID controller as

$$\tau_{\text{pid}}(t) = [K_i \ K_p \ K_d]e(t) \overset{\Delta}{=} Ke(t) \qquad (16)$$

where

$$K = [K_i \ K_p \ K_d].$$

Therefore, the tracking dynamics in (15) can be written as

$$\dot{e}(t) = (A + BK)e(t) + B(\Delta f(\eta) - \tau_f(\eta, \Xi))$$
$$+ B\tau_s(t) + B\tau_d(t) \qquad (17)$$

where $\tau_f(\eta, \Xi)$ is a fuzzy logic system control signal. The fuzzy logic system $\tau_f(\eta, \Xi)$ is proposed to approximate the uncertain term $\Delta f(\eta)$ where Ξ is initialized to be zero and adjusted during on-line operation. In this paper, since the fuzzy logic system $\tau_f(\eta, \Xi)$ is employed to approximate the uncertain term $\Delta f(\eta)$ in (17) to enhance the robust tracking, the fuzzy logic system $\tau_f(\eta, \Xi)$ in Fig. 1 is described in the following paragraphs.

The basic configuration of the fuzzy logic system for spacecraft systems attitude tracking control is shown in Fig. 1. The fuzzy logic system $\tau_f(\eta, \Xi)$ in this work performs a mapping from $U \in R^9$ to $V \in R^3$. Let $U = U_1 \times U_2 \times \cdots \times U_9$ where $U_i \subset R$, for $i = 1, 2, \ldots, 9$. The fuzzy rule base consists of a collection of fuzzy If-Then rules as follows:

Rule l: If η_1 is F_1^l, η_2 is $F_2^l, \ldots$ and η_9 is F_9^l

Then $\tau_f(\eta, \Xi)$ is G^l,   for $i = 1, 2, \ldots, M$

$$(18)$$

where

$$\eta = (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6, \eta_7, \eta_8, \eta_9)^T$$
$$= (\theta_1(t), \theta_2(t), \theta_3(t), \dot{\theta}_1(t), \dot{\theta}_2(t), \dot{\theta}_3(t), \ddot{\theta}_{d1}(t), \ddot{\theta}_{d2}(t), \ddot{\theta}_{d3}(t))^T \in U$$

and $\tau_f(\eta, \Xi) \in V \subset R^3$ are the input and output of the fuzzy logic system, respectively. In this paper, the fuzzy inference engine performs a mapping from fuzzy sets in $U \in R^9$ to fuzzy sets in R^9, based upon

the fuzzy If-Then rules in the fuzzy rule base and the compositional rule of inference.

REMARK 2   The angular acceleration is not available in most of the spacecraft applications. In our proposed method, the fuzzy logic system $\tau_f(\eta, \Xi)$ will be tuned to approximate $\Delta f(\eta)$ as close as possible; a sliding-mode control $\tau_s(t)$ is employed to eliminate the effect of approximation error. In this study, we use desired angular acceleration signals $\ddot{\theta}_d$ instead of $\ddot{\theta}$ to train fuzzy logic system. The discrepancy between $\ddot{\theta}$ and $\ddot{\theta}_d$ during transient conditions can cause large approximation errors, but the errors will converge with the time increasing. It is suitable for the regulation problems where $\ddot{\theta}_d = 0$.

LEMMA [10]   *If the fuzzy basis functions are defined as*

$$\zeta_{il}(\eta) = \frac{\prod_{j=1}^{9} \mu_{F_j^{il}}(\eta_j)}{\sum_{k=1}^{M} \left[\prod_{j=1}^{9} \mu_{F_j^{ik}}(\eta_j) \right]}$$

for $i = 1, 2, \ldots, 9$ and $l = 1, 2, \ldots, M$, then the fuzzy logic systems with center-average defuzzifier, product inference, and singleton fuzzifier for spacecraft tracking systems, are of the following form:

$$\tau_f(\eta, \xi) = \begin{bmatrix} \tau_1(\eta, \phi_1) \\ \tau_2(\eta, \phi_2) \\ \tau_3(\eta, \phi_3) \end{bmatrix} = \begin{bmatrix} \zeta_1^T(\eta) & 0 & 0 \\ 0 & \zeta_2^T(\eta) & 0 \\ 0 & 0 & \zeta_3^T(\eta) \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$
$$= \xi(\eta)\Xi \qquad (19)$$

where

$$\xi(\eta) = \begin{bmatrix} \zeta_1^T(\eta) & 0_{1 \times M} & 0_{1 \times M} \\ 0_{1 \times M} & \zeta_2^T(\eta) & 0_{1 \times M} \\ 0_{1 \times M} & 0_{1 \times M} & \zeta_3^T(\eta) \end{bmatrix}$$

$$\Xi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

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and $\phi_i = [\phi_{i1}, \phi_{i2}, \ldots, \phi_{iM}]^T$, $\zeta_i^T(\eta) = [\zeta_{i1}(\eta), \zeta_{i2}(\eta), \ldots, \zeta_{iM}(\eta)]^T$, for $i = 1, 2, \ldots, 9$ with that ϕ_{il} is the point at which the given membership function $\mu_{F_j^{ik}}(\phi_{il})$ achieves its maximum value, and we assume that $\mu_{F_j^{ik}}(\phi_{il}) = 1$.

In (17), the fuzzy logic system $\tau_f(\eta, \Xi)$ will be tuned to approximate $\Delta f(\eta)$ as closely as possible. In this paper, the fuzzy logic system $\tau_f(\eta, \Xi)$ is employed to approximate the uncertain $\Delta f(\eta)$ in (17) to enhance the robust tracking.

Let us define the optimal approximation parameter [10]

$$\Xi^* \overset{\Delta}{=} \arg \min_{\Theta \in \Omega_\Theta} \max_{\eta \in \Omega_\eta} \| \Delta f(\eta) - \xi(\eta) \Xi \| \tag{20}$$

where $\| \cdot \|$ denotes the Euclidean norm, i.e., $\|x\| = \sqrt{x^T x}$.

By the optimal approximation in (20), we obtain

$$\Delta f(\eta) = \xi(\eta) \Xi^* + \epsilon(\eta) \tag{21}$$

where $\epsilon(\eta) = [\epsilon_1(\eta), \epsilon_2(\eta), \epsilon_3(\eta)]^T$ denotes optimal approximation error with

$$|\epsilon_i(\eta)| \leq D_{\varepsilon_i}(\eta) \qquad \text{for} \quad i = 1, 2, 3$$

where $D_{\varepsilon_i}(\eta)$ denotes a known bound on the approximation error $\epsilon_i(\eta)$.

By substituting (21) into (17), we obtain

$$\dot{e}(t) = (A + BK)e(t) + B\xi(\eta)\tilde{\Xi} + B(\tau_s + \epsilon) + B\tau_d(t) \tag{22}$$

where

$$\tilde{\Xi} \overset{\Delta}{=} (\Xi^* - \Xi). \tag{23}$$

Let us consider the following H_∞ performance as follows [12]:

$$\int_0^{t_f} (e^T(t)Qe(t))dt \leq e^T(0)Pe(0) + \frac{1}{\gamma}\tilde{\Xi}^T(0)\tilde{\Xi}(0)$$
$$+ \rho^2 \int_0^{t_f} (\tau_d^T(t)\tau_d(t))dt \tag{24}$$

where $Q > 0$, $P = P^T > 0$ and ρ is a prescribed attenuation value which denotes the worst case effect of the external disturbances $\tau_d(t)$ on tracking error $e(t)$.

III. PID CONTROL WITH H_∞ TRACKING PERFORMANCE

In this section, we consider the effect of the external disturbance $\tau_d(t)$ on the tracking error $e(t)$ with the H_∞ performance. Let us define the Lyapunov function for (22) as

$$V(t) = e^T(t)Pe(t) + \frac{1}{\gamma}\tilde{\Xi}^T(t)\tilde{\Xi}(t). \tag{25}$$

From (24), we obtain

$$\int_0^{t_f} (e^T(t)Qe(t))dt \leq e^T(0)Pe(0) + \frac{1}{\gamma}\tilde{\Xi}^T(0)\tilde{\Xi}(0)$$

$$+ \int_0^{t_f} \left\{ e^T(t)Qe(t) + e^T(t)[(A + BK)^T P + P(A + BK)]e(t) \right.$$

$$+ \tilde{\Xi}^T(t)\xi^T(\eta)B^T Pe(t) + e^T(t)PB\xi(\eta)\tilde{\Xi}(t)$$

$$+ (\tau_s + \epsilon)^T B^T Pe(t) + e^T(t)PB(\tau_s + \epsilon) + \tau_d^T B^T Pe(t)$$

$$+ \left. e^T(t)PB\tau_d + \frac{1}{\gamma}\frac{d\tilde{\Xi}^T(t)}{dt}\tilde{\Xi}(t) + \frac{1}{\gamma}\tilde{\Xi}^T(t)\frac{d\tilde{\Xi}(t)}{dt} \right\}dt$$

$$\tag{26}$$

then we get the following main result.

THEOREM *For the uncertain spacecraft system (9), if the control*

$$\tau(t) = M_0(\theta)\left[\ddot{\theta}_d(t) + \left(K_i \tilde{\theta}_I(t) + K_p \tilde{\theta}(t) + K_d \frac{d\tilde{\theta}(t)}{dt} \right) \right.$$

$$\left. + \tau_s(t) - \xi(\eta)\Xi \right] + C_0(\theta, \dot{\theta})\dot{\theta} + G_0(\theta, \dot{\theta}) \tag{27}$$

with

$$\dot{\Xi}(t) = \gamma\xi^T(\eta)B^T Pe(t) \tag{28}$$

$$\tau_s(t) = -D_\epsilon(\eta)\text{sgn}(s) \tag{29}$$

where

$$s = B^T Pe(t)$$

$$D_\epsilon(\eta) = \begin{bmatrix} D_{\varepsilon_1}(\eta) & 0 & 0 \\ 0 & D_{\varepsilon_2}(\eta) & 0 \\ 0 & 0 & D_{\varepsilon_3}(\eta) \end{bmatrix}$$

and

$$\text{sgn}(s) = [\text{sgn}(s_1) \ \text{sgn}(s_2) \ \text{sgn}(s_3)]^T$$
$$= [\text{sgn}((B^T Pe(t))_1) \ \text{sgn}((B^T Pe(t))_2) \ \text{sgn}((B^T Pe(t))_3)]^T$$

where B_i denotes the ith column of B, $Q > 0$ is a weighting matrix, and $P = P^T > 0$ is the solution of the following linear matrix inequality:

$$(A + BK)^T P + P(A + BK) + \frac{1}{\rho^2}PBB^T P + Q < 0 \tag{30}$$

then the H_∞ control performance of (24) is guaranteed for a prescribed ρ^2.

PROOF Given $d\tilde{\Xi}(t)/dt = -\dot{\Xi}(t)$ from (23) and then from (26) we obtain

$$\int_0^{t_f} (e^T(t)Qe(t))dt \leq e^T(0)Pe(0) + \frac{1}{\gamma}\tilde{\Xi}^T(0)\tilde{\Xi}(0)$$

$$+ \int_0^{t_f} \left\{ e^T(t)Qe(t) + e^T(t)[(A+BK)^TP + P(A+BK)]e(t) \right.$$

$$+ (\tau_s + \epsilon)^T B^T Pe(t) + e^T(t)PB(\tau_s + \epsilon)$$

$$+ \tau_d^T(t)B^T Pe(t) + e^T(t)PB\tau_d(t)$$

$$+ \left(-\frac{1}{\gamma}\dot{\Xi}^T(t) + e^T(t)PB\xi(\eta)\right)\tilde{\Xi}(t)$$

$$+ \tilde{\Xi}^T(t)\left(-\frac{1}{\gamma}\dot{\Xi}(t) + \xi^T(\eta)B^T Pe(t)\right) \right\} dt.$$

By the update law in (28) and the sliding-mode control $\tau_s(t)$ in (29), we obtain

$$\int_0^{t_f} (e^T(t)Qe(t))dt \leq e^T(0)Pe(0) + \frac{1}{\gamma}\tilde{\Xi}^T(0)\tilde{\Xi}(0)$$

$$+ \int_0^{t_f} \left\{ e^T(t)[Q + (A+BK)^TP + P(A+BK) \right.$$

$$+ \left. \frac{1}{\rho^2}PBB^TP]e(t) + \rho^2\tau_d^T(t)\tau_d(t) \right\} dt. \quad (31)$$

Note that if

$$(A+BK)^TP + P(A+BK) + \frac{1}{\rho^2}PBB^TP + Q < 0 \tag{32}$$

then we obtain the following H_∞ tracking performance

$$\int_0^{t_f} (e^T(t)Qe(t))dt \leq e^T(0)Pe(0) + \frac{1}{\gamma}\tilde{\Xi}(0)^T\tilde{\Xi}(0)$$

$$+ \rho^2 \int_0^{t_f} (\tau_d^T(t)\tau_d(t))dt. \tag{33}$$

The sliding-mode control $\tau_s(t)$ can introduce a high-frequency signal, known as chattering phenomenons, to the plant which may excite unmodeled dynamics. To avoid this, the following smoothed control action is considered [15]. The function sgn(s_i) in the sliding-mode control is replaced by sat(s_i, σ) which is defined as follows:

$$\text{sat}(s_i, \sigma) = \begin{cases} \text{sgn}(s_i) & \text{if } |s_i| > \sigma \\ s_i/\sigma & \text{otherwise.} \end{cases}$$

REMARK 3 If the constrained problem due to the optimal approximation parameter Ξ^* that belongs to some preassigned compact set is considered, then additional tools concerned with the projection algorithm can be used to analyze this bounded problem. Suppose $\Omega_0 = \{\Xi : \Xi^T\Xi \leq \kappa\}$ and $\Omega_\Xi = \{\Xi : \Xi^T\Xi \leq \kappa + \delta\}$ where $\kappa > 0$ and $\delta > 0$. Then the

parameter law in (28) must be modified as follows [10]:

$$\dot{\Xi}(t) = \gamma\xi^T(\eta)B^T Pe(t) - U(\Xi(t)) \tag{34}$$

where

$$U(\Xi(t)) = \begin{cases} 0 & \text{if } \Xi^T(t)\Xi(t) \leq \kappa \quad \text{or} \quad (\Xi^T(t)\Xi(t) > \kappa \\ & \text{and} \quad \Xi^T(t)\xi^T(\eta)B^T Pe(t) \leq 0) \\ \gamma\dfrac{(\Xi^T\Xi - \kappa)\Xi^T\xi^T B^T Pe(t)}{\delta\Xi^T\Xi}\Xi & \text{otherwise} \end{cases}$$

where $\Xi(0) \in \Omega_0$ and $\Xi(t) \in \Omega_\Xi$.

Note that, by defining $W = P^{-1}$, (32) is equivalent to

$$W(A+BK)^T + (A+BK)W + \frac{1}{\rho^2}BB^T + WQW < 0. \tag{35}$$

With $Y = KW$ and by the Schur complements [14], (35) is equivalent to

$$\begin{bmatrix} WA^T + AW + (BY)^T + BY + \dfrac{1}{\rho^2}BB^T & W \\ W & -Q^{-1} \end{bmatrix} < 0. \tag{36}$$

By the change of variable $\nu = -1/\rho^2$ (where $\nu < 0$), (36) is equivalent the following LMI

$$\begin{bmatrix} WA^T + AW + (BY)^T + BY - \nu BB^T & W \\ W & -Q^{-1} \end{bmatrix} < 0. \tag{37}$$

To obtain better robust H_∞ tracking performance, we can minimize ν subject to (37) as the following eigenvalue problem (EVP)

$$\begin{aligned} & \min_{\{W,Y\}} && \nu \\ & \text{subject to} && W = W^T > 0, \quad \nu < 0 \text{ and } (37) \end{aligned}. \tag{38}$$

Design Procedure:
Step 1 Select membership functions and fuzzy rules in (18).
Step 2 Given the parameters $D_{\varepsilon_1}(\eta)$, $D_{\varepsilon_2}(\eta)$, $D_{\varepsilon_3}(\eta)$ and σ.
Step 3 Given an initial attenuation level ρ, select weighting matrix Q and $\gamma > 0$ according to the design purpose.
Step 4 Solve the EVP in (38) to obtain P (thus $P = W^{-1}$, the PID control gain $K = YW^{-1}$ can also be obtained).
Step 5 Decrease ρ and repeat Steps 4–5 until P cannot found.
Step 6 Construct the controller in (27).

IV. SIMULATION EXAMPLES

To substantiate the performance of the robust adaptive PID attitude tracking control design,

simulations of the attitude control on the spacecraft system are considered in this section. The inertia matrix J is assumed to be

$$J = J_0 + \Delta J$$

where

$$J_0 = \begin{bmatrix} 126.98 & -1.87 & 3.38 \\ -1.87 & 116.63 & -2.40 \\ 3.38 & -2.40 & 209.36 \end{bmatrix}$$

$$\Delta J = \begin{bmatrix} \Delta J_{11} & \Delta J_{12} & \Delta J_{13} \\ \Delta J_{21} & \Delta J_{22} & \Delta J_{23} \\ \Delta J_{31} & \Delta J_{32} & \Delta J_{33} \end{bmatrix}$$

where

$$\Delta J_{11} = 0.2 \, \text{sign}(\sin(0.1t))$$

$$\Delta J_{22} = -0.2 \, \text{sign}(\cos(0.3t))$$

$$\Delta J_{33} = 0.9 \, \text{sign}(\sin(0.18t)\sin(0.25t))$$

$$\Delta J_{12} = \Delta J_{21} = 0.3 \, \text{sign}(\cos(0.15t))$$

$$\Delta J_{13} = \Delta J_{31} = 0.2 \, \text{sign}(\sin(0.1t)\cos(0.2t))$$

$$\Delta J_{23} = \Delta J_{32} = -0.3 \, \text{sign}(\sin(0.1t)).$$

To illustrate, the robust adaptive PID control design is achieved by the following steps.

Step 1   Because there are three outputs of fuzzy logic system with each corresponding to 7 fuzzy rules and 9 state variables, the following 189 membership functions are selected:

$$\mu_{F_j^{i1}} = \exp[-(\eta_j - 3a_j)^2]$$

$$\mu_{F_j^{i2}} = \exp[-(\eta_j - 2a_j)^2]$$

$$\mu_{F_j^{i3}} = \exp[-(\eta_j - a_j)^2]$$

$$\mu_{F_j^{i4}} = \exp[-(\eta_j)^2]$$

$$\mu_{F_j^{i5}} = \exp[-(\eta_j + a_j)^2]$$

$$\mu_{F_j^{i6}} = \exp[-(\eta_j + 2a_j)^2]$$

$$\mu_{F_j^{i7}} = \exp[-(\eta_j + 3a_j)^2]$$

for $i = 1,2,3$ and $j = 1,2,\ldots,9$ where $a_1 = a_2 = a_3 = \pi/10$, $a_4 = a_5 = a_6 = \pi/20$ and $a_7 = a_8 = a_9 = \pi/30$.

The fuzzy rules in the following form are included in the fuzzy rule bases.

Rule $i1$:   If η_1 is $F_1^{i1},\ldots,$ and η_9 is F_9^{i1}

Then $\tau_i(\eta, \Xi)$ is G^{i1}

$$\vdots$$

Rule $i7$:   If η_1 is $F_1^{i7},\ldots,$ and η_9 is F_9^{i7}

Then $\tau_i(\eta, \Xi)$ is G^{i7}

for $i = 1, 2, 3$.

Denote

$$D = \sum_{k=1}^{7} \prod_{j=1}^{9} \mu_{F_j^{ik}}(\eta_j)$$

and we have

$$\xi(\eta) = \begin{bmatrix} \zeta_1^T(\eta) & 0 & 0 \\ 0 & \zeta_2^T(\eta) & 0 \\ 0 & 0 & \zeta_3^T(\eta) \end{bmatrix}$$

$$\Xi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix}$$

where

$$\phi_i = [\phi_{i1}, \phi_{i2}, \ldots, \phi_{i7}]^T$$

$$\zeta_i^T(\eta) = \left[\frac{\prod_{j=1}^{9} \mu_{F_j^{i1}}(\eta_j)}{D} \cdots \frac{\prod_{j=1}^{9} \mu_{F_j^{i7}}(\eta_j)}{D} \right]^T$$

for $i = 1,2,3$.

For the convenience of simulations, we choose $\Xi(0) = 0$ and $\gamma = 1$.

Step 2   We specify the parameters for sliding-mode control with $D_{\varepsilon_1}(\eta) = D_{\varepsilon_2}(\eta) = D_{\varepsilon_3}(\eta) = 0.5$ and $\sigma = 0.1$.

Step 3   Select $Q = 0.0001 \times I_9$ and $\gamma = 0.3$.

Step 4–5   We obtain $\rho_{\min} = 0.5774$ and the PID control gain

$$K = \begin{bmatrix} -0.0162 & 0 & 0 & -0.1429 \\ 0 & -0.0162 & 0 & 0 \\ 0 & 0 & -0.0162 & 0 \\ & & & \\ 0 & 0 & -0.5491 & 0 & 0 \\ -0.1429 & 0 & 0 & -0.5491 & 0 \\ 0 & -0.1429 & 0 & 0 & -0.5491 \end{bmatrix}.$$

The simulation results are shown in Figs. 2–4. The trajectories of $\theta(t)$ and $\theta_d(t)$ are shown in Fig. 2. The disturbance inputs are shown in Fig. 3. The control inputs are shown in Fig. 4. From the simulation results, we observe that the adaptive robust H_∞ PID tracking control design has good performance.

In this paper, we compare our results with that in [7], [8], and [16] where the spacecraft robust attitude tracking design is obtained based on the nonlinear H_∞ control theory. Some comparisons are given as follows.

1) The choice of PID parameters in [16] is complex and varies from case to case. Our methods are simple and the PID parameters can be obtained systematically.

2) In [7], [8], and [16], the robust tracking control problem for spacecraft systems is characterized in
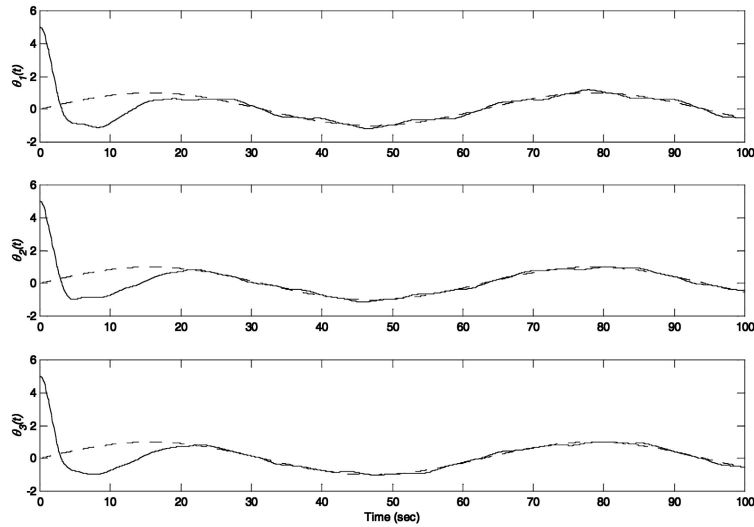
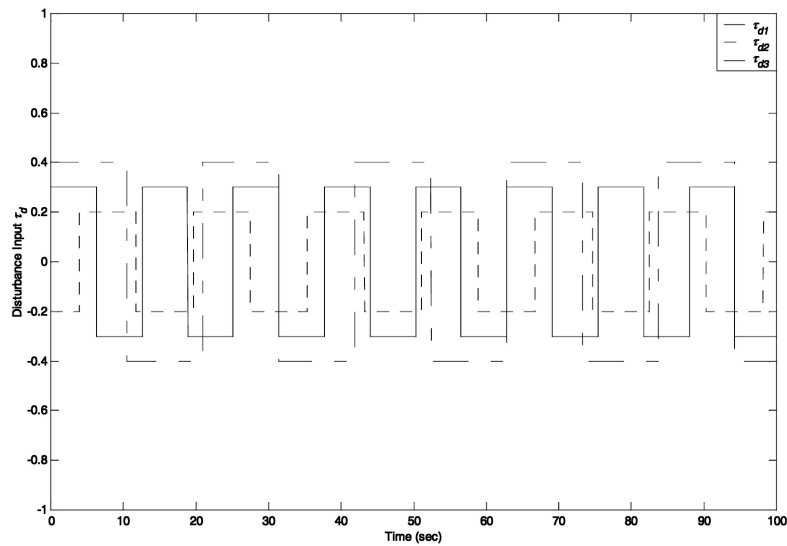Fig. 2. Trajectories of $\theta(t)$ and $\theta_d(t)$.

Fig. 3. Disturbances input.

terms of the Hamilton-Jacobi partial differential equations. Until now, it has been very difficult to solve the Hamilton-Jacobi partial differential equations either analytically or numerically. In our methods, the robust adaptive PID tracking control problems are characterized in terms of EVP. The EVP can be efficiently solved by the LMI toolbox in MATLAB. The proposed methods are simple and the PID parameters can be obtained systematically.

3) In practice, parameters in the system are subject to the uncertainties and unmodeled dynamics. When the system's parameters are varying or uncertain, the performance is degraded [16]. In our attitude tracking design, an adaptive fuzzy scheme is employed to approximate the parameters' uncertainties.

Therefore, the proposed design method is appropriate for the robust adaptive PID tracking control design of spacecraft systems with uncertainties and external disturbances.

V. CONCLUSIONS

In this paper, a PID-type controller incorporating a fuzzy logic system for robust H_∞ tracking performance is developed in spacecraft systems under plant uncertainties and external disturbances. The fuzzy logic system is used to approximate the plant uncertainties. The robust H_∞ tracking performance is related to the attenuation property with respect to the external disturbances.

Sufficient conditions are developed in terms of LMI formulation. The robust PID tracking control problems of the uncertain spacecraft systems are characterized in terms of EVP. The EVP can be efficiently solved by the LMI toolbox in Matlab. The proposed methods are simple and the PID control gains can be obtained systematically. Simulation results indicate that the desired tracking performance for the robust control schemes of the uncertain
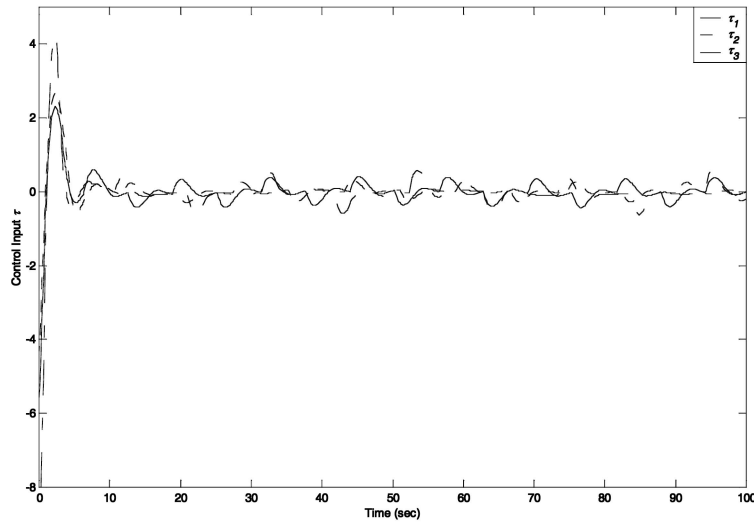
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Fig. 4.   Control input.

spacecraft systems can be achieved using the proposed methods.

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